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## QCD Sum Rules and Chiral Symmetry Breaking

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### Abstract

We investigate, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We describe how such matching yields predictions on induced condensates which reproduce the observed value of the strong  $\pi NN$  coupling constant. Our result indicate that the observed large  $\pi NN$  coupling comes primarily from the nonperturbative induced-condensate effect. On the other hand, the prediction on the parity-violating weak  $\pi NN$  coupling,  $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$  is in good agreement with the value ( $\sim 2 \times 10^{-7}$ ) obtained by Adelberger and Haxton from an overall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent null experiment. Altogether, we wish to argue that the proposal of matching onto a chirally broken effective theory should enable us to treat, in a quantitative and consistent manner, the various nonleptonic strong and weak processes involving Goldstone bosons.

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## I. Introduction

The problem of strong interaction physics has been with us for more than half a century, despite the fact that the very nature of the problem has been changing with the so-called “underlying theory” which nowadays is taken universally to be quantum chromodynamics or QCD. Although the asymptotically free nature of QCD allows us to test the candidate theory at high energies, the nonperturbative feature dominates at low energies so that it remains almost impossible to solve problems related to hadrons or nuclei, except perhaps through lattice simulations but it is unlikely that, through the present-day computation power which is still not quite adequate, simulations would give us most of what we wish to know. As a result, it would be useful to evaluate the merits of using the method of QCD sum rules [1] to bridge between the description of the hadron properties and what we may expect from QCD.

The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green’s functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv \langle 0 | T(q_i^a(x)\bar{q}_j^b(0)) | 0 \rangle, \quad (1)$$

carry all the difficulties inherent in the theory. Higher-order condensates, such as a four-quark condensate,

$$\langle 0 | T(\bar{\psi}(z)\gamma_\mu\psi(z)q_i^a(x)\bar{q}_j^b(0)) | 0 \rangle,$$

with  $\psi(z)$  also labeling a quark field, represent an infinite series of unknowns unless some useful ways for reduction can be obtained. As the vacuum,  $| 0 \rangle$ , is highly nontrivial, there is little reason to expect that Wick’s theorem (of factorization), as obtained for free quantum field theories, is still of validity. Thus, we must look for alternative routes in order to obtain useful results or relations.

Accordingly, it might be useful, before embarking on the subject of chiral symmetry breaking in relation to QCD sum rules, to detour slightly by considering the feasibility of working directly with the various matrix elements (rather than the operators). To this end, we note that, at the classical level, we may start with the QCD lagrangian [2],

$$\begin{aligned}\mathcal{L}_{QCD} = & \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2 \\ & - \bar{\eta}^a \partial_\mu (\partial^\mu \delta^{ac} + g f^{abc} A^{\mu b}) \eta^c \\ & + \bar{\psi} \{ i\gamma^\mu (\partial_\mu + ig \frac{\lambda^a}{2} A_\mu^a) - m \} \psi,\end{aligned}\tag{2}$$

with the gauge-fixing and ghost terms (i.e., the second and third terms) shown explicitly. We then have the equations for interacting fields,

$$\{ i\gamma^\mu (\partial_\mu + ig \frac{\lambda^a}{2} A_\mu^a) - m \} \psi = 0;\tag{3}$$

$$\partial^\nu G_{\mu\nu}^a - 2g f^{abc} G_{\mu\nu}^b G_\nu^c + g \bar{\psi} \frac{\lambda^a}{2} \gamma_\mu \psi = 0,\tag{4}$$

which may be re-interpreted as the equations of motion for *quantized* interacting fields. As the (standard) rule for quantization, the equal-time (anti-)commutators among these quantized interacting fields are identical to those among non-interacting quantized fields.

Allowing the operator  $\{ i\gamma^\mu \partial_\mu - m \}$  to act on the matrix element defined by Eq. (1), we obtain, making use of what we have just stated about interacting field equations and (canonical) quantization rules,

$$\{ i\gamma^\mu \partial_\mu - m \}_{ik} iS_{kj}^{ab}(x) = i\delta^4(x) \delta^{ab} \delta_{ij} + \langle 0 | T(\{ g \frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x) \}_i \bar{q}_j^b(0)) | 0 \rangle .\tag{5}$$

This equation can be solved by splitting the propagator into a singular, perturbative part and a nonperturbative part:

$$iS_{ij}^{ab}(x) = iS_{ij}^{(0)ab}(x) + i\tilde{S}_{ij}^{ab}(x),\tag{6}$$

with

$$iS_{ij}^{(0)ab}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} iS_{ij}^{(0)ab}(p), \quad (7a)$$

$$iS_{ij}^{(0)ab}(p) = \delta^{ab} \frac{i(\not{p} + m)_{ij}}{p^2 - m^2 + i\epsilon}. \quad (7b)$$

Accordingly, we have

$$\{i\gamma^\mu \partial_\mu - m\}_{ik} i\tilde{S}_{kj}^{ab}(x) = \langle 0 | T(\{g \frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x)\}_i \bar{q}_j^b(0)) | 0 \rangle. \quad (8)$$

With the fixed-point gauge,

$$A_\mu^n(x) = -\frac{1}{2} G_{\mu\nu}^n x^\nu + \dots, \quad (9)$$

we may then solve the nonperturbative part  $i\tilde{S}_{ij}^{ab}(x)$  as a power series in  $x^\mu$ ,

$$\begin{aligned} i\tilde{S}_{ij}^{ab}(x) = & -\frac{1}{12} \delta^{ab} \delta_{ij} \langle \bar{q}q \rangle + \frac{i}{48} m (\gamma_\mu x^\mu)_{ij} \delta^{ab} \langle \bar{q}q \rangle \\ & + \frac{1}{192} \langle \bar{q}g\sigma \cdot Gq \rangle \delta^{ab} x^2 \delta_{ij} + \dots, \end{aligned} \quad (10)$$

The first term is the integration constant which defines the so-called “quark condensate”, while the mixed quark-gluon condensate appearing in the third term arises because of Eqs. (8) and (9). It is obvious that the series (10) is a short-distance expansion, which converges for small enough  $x_\mu$ .

A practitioner would recognize immediately that Eq. (10) is just the standard quark propagator cited in most papers in QCD sum rules. The approach which we suggest here is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as Eq. (5), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that the nontriviality of the vacuum  $| 0 \rangle$  is observed at every step – a central issue in relation to QCD. Although Eq. (10) seems to be standard (and natural), our approach makes it intuitive and relatively

straightforward to work out the problem to an arbitrary order as well as to treat many other (propagator-like) matrix elements such as those to be used later in the paper.

In light of the fact that the vacuum  $|0\rangle$  is nontrivial (a fact which renders useless many lessons from perturbative quantum field theory), the route of starting directly from the interacting field equations, augmented by the canonical quantization rule, may be pursued seriously. The method, in which renormalizations need to be defined order by order in a consistent manner, may be contrasted with the usual path-integral formulation in which Green's functions may be derived, in an elegant manner, from the generating functional. The method should be seriously exploited mainly because, just like what we have done in the above, it might offer some new insights for the problem at hand, i.e. QCD.

As another important exercise, we may split the gluon propagator into the singular, perturbative part and the nonperturbative part. For the nonperturbative part, our interacting-field-equation-based approach yields

$$\begin{aligned}
& g^2 < 0 | : G_{\mu\nu}^n(x) G_{\alpha\beta}^m(0) : | 0 \rangle \\
& = \frac{\delta^{nm}}{96} \langle g^2 G^2 \rangle (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \\
& \quad - \frac{\delta^{nm}}{192} \langle g^3 G^3 \rangle \{ x^2 (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) - g_{\mu\alpha} x_\nu x_\beta + g_{\mu\beta} x_\nu x_\alpha - g_{\nu\beta} x_\mu x_\alpha + g_{\nu\alpha} x_\mu x_\beta \} \\
& \quad + O(x^4),
\end{aligned} \tag{11}$$

with

$$\langle g^2 G^2 \rangle \equiv \langle 0 | : g^2 G_{\mu\nu}^n(0) G^{n\mu\nu}(0) : | 0 \rangle, \tag{12a}$$

$$\langle g^3 G^3 \rangle \equiv \langle 0 | : g^3 f^{abc} g^{\mu\nu} G_{\mu\alpha}^a(0) G^{b\alpha\beta}(0) G_{\beta\nu}^c(0) : | 0 \rangle. \tag{12b}$$

Again, the first term in Eq. (11) is the integration constant for the differential equation satisfied by the gluon propagator. Note that inclusion of the gluon condensate  $\langle g^2 G^2 \rangle$  in Eq. (11) is still standard but the triple gluon condensate  $\langle g^3 G^3 \rangle$  is a new entry required

by the interacting field equation (4). Our approach indicates when condensates of entirely new types should be introduced as we try to perform operator-product expansions to higher dimensions.

The method of QCD sum rules may now be regarded as the various approaches in which we try to exploit the roles played by the quark and gluon condensates which enter Eqs. (10) and (11) for problems involving hadrons. To this end, we wish to set up a quantum field theory in which the vacuum is nontrivial, while leaving some condensates as parameters to be determined *via* first principle, such as from lattice simulations of QCD. The aim is such that, once the fundamental condensate parameters are given, one can make predictions on the various physical quantities associated with hadrons. Of course, the method of QCD sum rules in its present broad context has its origin in [1].

## II. Induced Condensates and the Effective Chiral Quark Theory

The quark condensate which enters the various QCD sum rules, such as the Belyaev-Ioffe nucleon mass sum rule [3] and the sum rules [4] for the nucleon axial couplings, is perhaps the best known parameter among all condensates:

$$a \equiv -(2\pi)^2 \langle \bar{q}q \rangle \simeq 0.546 \text{ GeV}^3, \quad (13)$$

which is the order parameter in the chirally broken phase of QCD. It is related to the pion decay constant  $f_\pi$  with  $4\pi f_\pi$  being the scale governing the convergence property of the expansion in chiral perturbation theory. One point which is worth mentioning at this juncture is that, as we vary  $Q^2$  in probing the hadron substructure, the nonzero condensate as listed in Eqs. (13) does not disappear suddenly although the effects may become negligibly small for sufficiently large  $Q^2$  — unlike the situation when one varies the temperature  $T$  and sees some phase transition at some critical temperature  $T_c$ . As long as we submit to the picture where the condensate (13) differs from zero, it is important to recognize that

we are working with a condensated phase of QCD – which already differs from the naive QCD (without condensation). It is also natural to ask what other effects such condensation (phase transition) brings to light. For instance, it is believed that the spontaneous symmetry breaking leading to sizable condensates also gives rise to the dynamically-generated or constituent mass to a quark. Such spontaneous symmetry breaking refers to the breaking of  $SU(N_f)_L \times SU(N_f)_R$  chiral symmetry for QCD of  $N_f$  flavors of almost massless quarks into the  $SU(N_f)_V$  symmetry which, according to Goldstone theorem, must give rise to  $N_f^2 - 1$  almost massless Goldstone bosons, pseudoscalar bosons in the present case (with pions and other pseudoscalar mesons as the natural candidate). If we restrict ourselves to the world of up and down quarks (as we shall do in the rest of this paper), there is little reason to reject the role played by Goldstone pions while accepting quite large empirical values for the various condensates.

In light of the above consideration, we are led to investigate, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We immediately recognize that such proposal has many ramifications for the method of QCD sum rules. In any event, the proposal should first be checked out to leading order in the Goldstone fields – such as very small Goldstone pion fields. A natural problem for such investigation is the determination of the strong  $\pi NN$  coupling, for which we shall show that the proposal indeed yields predictions on induced condensates which are just needed to reproduce the observed value and that such large  $\pi NN$  coupling comes primarily from the nonperturbative induced-condensate effect. A closely related question is the parity-violating (p.v.) weak  $\pi NN$  coupling which is considerably more complicated since it involves calculation (and regularization) of three-loop diagrams. In any event, it is of importance to use the determination of the  $\pi NN$  couplings as the first testing ground for the proposal. This is what we wish to do in the present paper, while having to put off investigations of some other serious issues for the future.

What should we use while still insisting on using quarks and gluons as the effective degrees of freedom (DOF's)? A plausible choice is to accept the chiral lagrangian of Weinberg [5] and Georgi [6] as the candidate effective theory, in which Goldstone bosons arising from chiral symmetry breaking couple to quarks (and indirectly to gluons) but such couplings should vanish in the long-wavelength limit. The choice may not be unique as far as identification of Goldstone pions (with physical pions) is concerned, but many different choices may turn out to be equivalent (as some people believe to be the case).

On the other hand, it is often said that the long-distance realization, or the low-energy effective theory, of QCD is chiral perturbation theory ( $\chi$ PT) [7], in which Goldstone bosons are the only effective DOF's. As we go up in the energy scale, we must eventually end up in having to deal directly with QCD itself. Thus, it is also natural to anticipate that, at some intermediate energy scale, the two languages, namely  $\chi$ PT and QCD, can be matched onto each other or, in somewhat loose terms, the DOF's of the two theories couple in a way dictated by symmetries, yielding again the effective chiral quark theory [5,6]. The resultant theory is unique if symmetries dictates completely the interactions, as suspected to be so in the present case.

Therefore, there leaves little room for what we may choose for the effective chiral quark theory [5,6], onto which we shall try to map the world of hadrons. As the primary example of this paper, we consider the breaking of the  $SU(2)_L \times SU(2)_R$  chiral symmetry into the vector  $SU(2)$  symmetry. The effective chiral quark theory is then specified, to dimension six, by [5]

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \{ i\gamma^\mu \partial_\mu - \frac{1}{4f_\pi^2} \gamma^\mu (\partial_\mu \vec{\pi}) \times \vec{\pi} \cdot \vec{\tau} \} \psi \\ & - \frac{g_A}{2f_\pi} \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi \cdot \partial_\mu \vec{\pi} - m \bar{\psi} \psi + \frac{1}{2} (1 - \frac{\vec{\pi}^2}{4f_\pi^2}) \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \\ & - \frac{\mu^2}{2} (1 - \frac{\vec{\pi}^2}{4f_\pi^2}) \vec{\pi}^2 - \bar{\psi} \Gamma_\alpha \psi \bar{\psi} \Gamma^\alpha \psi, \end{aligned} \tag{14}$$

to which the gluon DOF's should be added as in Eq. (2), but with considerably weaker



coupling constant.

The method of QCD sum rules starts with a choice of some correlation function which may be evaluated at the quark level on the one hand (the so-called “left-hand side” or LHS) while may also be interpreted at the hadron level (the so-called “right-hand side” or RHS). To study the nucleon properties, for instance, we may choose

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T[\eta_p(x) \bar{\eta}_p(0)] | 0 \rangle , \quad (15a)$$

with the usual choice [8] of the current  $\eta(x)$

$$\eta_p(x) = \epsilon^{abc} [u^{aT}(x) C \gamma_\mu u^b(x)] \gamma^5 \gamma^\mu d^c(x). \quad (15b)$$

Here  $C$  is the charge conjugation operator and  $\langle 0 | \eta(0) | N(p) \rangle = \lambda_N u_N(p)$  with  $\lambda_N$  the amplitude for overlap and  $u_N(p)$  the nucleon Dirac spinor (normalized such that  $\bar{u}_N(p) u_N(p) = 2m_N$ ). And we may work out both the LHS and RHS sides and obtain the now well-known QCD sum rules [3, 4].

This is “duality” in the sense that some quantity assumes suitable meaning both at the quark level and at the hadron level. As elucidated in detail so far, we wish to investigate the proposal that, in such duality picture, the language to be used at the quark level is the effective chiral quark theory, rather than QCD in the chirally symmetric phase (in which Goldstone bosons do not enter as relevant DOF’s). Since chiral symmetry breaking has already taken place with the order parameter as given by Eq. (13), it appears quite natural that we should be trying to match the world of hadrons to a world in which we could talk about not only quarks and gluons but also sizable basic condensates.

To proceed further, we note [9] that, on an empirical ground, the quark propagator is modified in the presence of sufficiently small pion fields:

$$\begin{aligned} i\delta S^{ab}(x) = & - \frac{i\vec{\tau} \cdot \vec{\pi}}{4\pi^2 x^2} g_{\pi q} \gamma^5 \delta^{ab} \\ & + \frac{i}{24} \vec{\tau} \cdot \vec{\pi} g_{\pi q} \chi_\pi \langle \bar{q}q \rangle \delta^{ab} \gamma^5 , \\ & - \frac{i}{3 \cdot 2^7} m_0^\pi \langle \bar{q}q \rangle g_{\pi q} \vec{\tau} \cdot \vec{\pi} x^2 \gamma^5 , \end{aligned} \quad (16)$$

with

$$< \bar{q} i \tau_j \gamma_5 q >_\pi \equiv \chi_\pi g_{\pi q} \pi_j < \bar{q} q > \quad (17a)$$

$$< \bar{q} i \gamma_5 \tau_j \sigma \cdot G q >_\pi \equiv m_0^\pi g_{\pi q} \pi_j < \bar{q} q > . \quad (17b)$$

Here  $g_{\pi q}$  is the pion-quark coupling, which has been introduced on a purely empirical ground. The susceptibility  $\chi_\pi$  enters in the evaluation of both strong and weak pion-nucleon coupling constants, while  $m_0^\pi$  enters only for the weak one.

We may begin by working in the limit of massless Goldstone pions ( $\mu^2 = 0$ ). In this limit, all the terms involving Goldstone bosons must contain the derivative  $\partial_\mu \vec{\pi}$ , as shown by the lagrangian of Eq. (14). Denote these terms altogether by  $\delta\mathcal{L}$ . For the primary purpose of the present paper, it is sufficient to treat such pion fields as some small classical fields. Thus, we obtain the modification to the quark propagator as defined by Eq. (1).

$$\begin{aligned} i\delta S_{ij}^{ab}(x) &= < 0 | T(i \int d^4 z \delta\mathcal{L}(z) q_i^a(x) \bar{q}_j^b(0)) | 0 > \\ &= -i \int d^4 z \partial^\mu \vec{\pi}(z) \cdot < 0 | T(\vec{A}_\mu(z) q_i^a(x) \bar{q}_j^b(0)) | 0 > \\ &= i \frac{g_A}{2f_\pi} \int d^4 z \vec{\pi}(z) \cdot \partial^\mu < 0 | T(\vec{A}_\mu(z) q_i^a(x) \bar{q}_j^b(0)) | 0 >, \end{aligned} \quad (18)$$

with  $\vec{A}_\mu(x)$  the isovector axial vector current as given by

$$\vec{A}_\mu = \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \psi - \frac{2f_\pi}{g_A} (1 - \frac{\vec{\pi}^2}{4f_\pi^2}) \partial^\mu \vec{\pi} - \frac{1}{2g_A f_\pi} \bar{\psi} \gamma^\mu \vec{\tau} \times \vec{\pi} \psi. \quad (19)$$

Note that the second equality in Eq. (18) follows since we shall treat pions fields  $\vec{\pi}(x)$  as some small classical field. It is straightforward to show that PCAC holds:

$$\partial_\mu \vec{A}^\mu = \frac{2f_\pi}{g_A} \mu^2 \vec{\pi} + O(\frac{1}{f_\pi}). \quad (20)$$

A little algebra yields, in the limit with  $\mu^2 = 0$ ,

$$\begin{aligned} i\delta S_{ij}^{ab}(x) &= -i \frac{g_A}{2f_\pi} \vec{\pi}(x) \cdot < 0 | T(\{\gamma_5 \vec{\tau} q(x)\}_i^a \bar{q}_j^b(0)) | 0 > \\ &\quad - i \frac{g_A}{2f_\pi} \vec{\pi}(0) \cdot < 0 | T(q_i^a(x) \{\bar{q}(0) \gamma_5 \vec{\tau}\})_j^b | 0 > . \end{aligned} \quad (21)$$

This is in fact a notable case in which the expectation value of the product of some four-quark operators can be reduced to an expression involving only two-quark operators – via the application of PCAC.

The two propagators appearing in Eq. (21) can be solved in a way introduced in the previous section for the propagator  $iS_{ij}^{ab}(x)$ . Namely, we use Eq. (3) and equal-time anti-commutators and obtain

$$\begin{aligned} & \{i\gamma^\mu \partial_\mu + m\}_{ik} < 0 | T(\{\gamma_5 \vec{\tau} q(x)\}_k^a \bar{q}_j^b(0)) | 0 > \\ &= -i\delta^4(x) \delta^{ab} \gamma^5 \vec{\tau} - < 0 | T(\{\gamma^5 \vec{\tau} g \frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x)\}_i^a \bar{q}_j^b(0)) | 0 >, \end{aligned} \quad (22a)$$

$$\begin{aligned} & \{i\gamma^\mu \partial_\mu - m\}_{ik} < 0 | T(q_k^a(x) \{\bar{q}(0) \gamma_5 \vec{\tau}\}_j^b) | 0 > \\ &= i\delta^4(x) \delta^{ab} \gamma^5 \vec{\tau} + < 0 | T(\{g \frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x)\}_i^a \{\bar{q}(0) \gamma^5 \vec{\tau}\}_j^b(0)) | 0 >. \end{aligned} \quad (22b)$$

The solutions to Eqs. (22a) and (22b) are

$$\begin{aligned} & < 0 | T(\{\gamma_5 \vec{\tau} q(x)\}_i^a \bar{q}_j^b(0)) | 0 > \\ &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \delta^{ab} \frac{i[(-\not{p} + m) \gamma^5 \vec{\tau}]_{ij;mn}}{p^2 - m^2 + i\epsilon} - \frac{1}{12} \delta^{ab} \gamma^5 \vec{\tau} < \bar{q} q > - \frac{i}{48} m \gamma_\mu x^\mu \gamma^5 \vec{\tau} \delta^{ab} < \bar{q} q > \\ &+ \frac{1}{192} < \bar{q} g \sigma \cdot G q > \delta^{ab} \gamma^5 \vec{\tau} x^2 + \dots, \end{aligned} \quad (23a)$$

$$\begin{aligned} & < 0 | T(q_i^a(x) \{\bar{q}(0) \gamma_5 \vec{\tau}\}_j^b) | 0 > \\ &= \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \delta^{ab} \frac{i[(\not{p} + m) \gamma^5 \vec{\tau}]_{ij;mn}}{p^2 - m^2 + i\epsilon} - \frac{1}{12} \delta^{ab} \gamma^5 \vec{\tau} < \bar{q} q > + \frac{i}{48} m \gamma_\mu x^\mu \gamma^5 \vec{\tau} \delta^{ab} < \bar{q} q > \\ &+ \frac{1}{192} < \bar{q} g \sigma \cdot G q > \delta^{ab} \gamma^5 \vec{\tau} x^2 + \dots, \end{aligned} \quad (23b)$$

We caution that the quark mass  $m$  in these equations is the one appearing in the effective chiral quark theory, Eq. (14), rather than that in the bare QCD, Eq. (2) [the chirally symmetric phase of QCD].

Substituting Eqs. (23a) and (23b) back into Eq. (21) and treating  $\vec{\pi}$  as a small constant field, we obtain

$$\begin{aligned}
& i\delta S^{ab}(x) \\
& = + \frac{g_A m}{f_\pi} \vec{\tau} \cdot \vec{\pi} \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \frac{\delta^{ab} \gamma_5}{p^2 - m^2 + i\epsilon} \\
& \quad + i \frac{g_A}{f_\pi} \vec{\tau} \cdot \vec{\pi} \left\{ \frac{1}{12} \langle \bar{q}q \rangle \delta^{ab} \gamma_5 - \frac{1}{192} \langle \bar{q}g\sigma \cdot Gq \rangle \delta^{ab} x^2 \gamma_5 \right\}.
\end{aligned} \tag{24}$$

In the effective chiral quark theory [6], we use  $m \approx 0.35 \text{ GeV}$ . As we usually take  $p^2 \sim 1 \text{ GeV}^2$ , we may neglect  $m^2$  as compared to  $p^2$  in the first term of Eq. (24) so that the expression (24) reduces to Eq. (16) with

$$g_{\pi q} = -\frac{g_A m}{f_\pi}, \tag{25a}$$

$$\chi_\pi g_{\pi q} = \frac{2g_A}{f_\pi}, \tag{25b}$$

$$m_0^\pi \langle \bar{q}q \rangle g_{\pi q} = \frac{2g_A}{f_\pi} \langle \bar{q}g\sigma \cdot Gq \rangle. \tag{25c}$$

Accordingly, the pion-quark coupling  $g_{\pi q}$  and the induced condensates as characterized by the susceptibilities  $\chi_\pi$  and  $m_0^\pi$  are completely determined when the connection with the effective chiral quark theory is made. Numerically, we may use [6]

$$f_\pi = 93 \text{ MeV}, \quad g_A = 0.7524, \tag{26a}$$

so that

$$g_{\pi q} \sim -2.83, \quad \chi_\pi a = -\frac{2a}{m} \approx -3.14 \text{ GeV}^2. \tag{26b}$$

Note that we also have  $m_0^\pi = \frac{2m_0^2}{m}$  with  $\langle \bar{q}g\sigma \cdot Gq \rangle \equiv -m_0^2 \langle \bar{q}q \rangle$  but we shall discuss the parameter  $m_0^2$  later in the paper. We re-iterate again that, since the whole derivation is made by making connection with an effective chiral quark theory, it is natural to adopt the constituent quark mass in Eqs. (26).

### III. Pion-Nucleon Couplings

To investigate the proposal that the matching is done between the hadron world and the effective chiral quark theory [5,6], we wish to consider the evaluation of both the strong and weak  $\pi NN$  coupling constants. Such evaluation, by itself being of fundamental importance, can readily be generalized to treat a large number of physical processes involving Goldstone bosons, such as nonleptonic decays of hyperons or heavy mesons ( $\Lambda \rightarrow N\pi$ ,  $D \rightarrow K\pi$ , etc.).

To begin, it is useful to recall the Belyaev-Ioffe nucleon mass sum rule, taking into account the quark mass terms [3, 10],

$$\begin{aligned} \frac{1}{8}M_B^6 L^{-4/9} E_2 + \frac{1}{32} \langle g_c^2 G^2 \rangle M_B^2 L^{-4/9} E_0 + \frac{1}{6} a^2 L^{4/9} - \frac{a^2 m_0^2}{24 M_B^2} L^{-2/27} \\ - \frac{ma M_B^2}{4} L^{-4/9} E_0 - \frac{mam_0^2}{8} L^{-26/27} = \beta_N^2 e^{-M^2/M_B^2}, \end{aligned} \quad (27)$$

with  $\beta_N^2 \equiv \frac{1}{4}(2\pi)^4 |\lambda_N|^2$ . The factors containing  $L$ ,  $L = 0.621 \ln(10M_B)$ , give the evolution in  $Q^2$  arising from the anomalous dimensions, and the  $E_i(M_B^2)$  functions take into account excited states to ensure the proper large- $M_B^2$  behavior [3, 4].

By analyzing the observed mass splittings in heavy quarkonia, Narison [11] extracted an updated average value on the gluon condensate:

$$\langle g_c^2 G^2 \rangle = (8.9 \pm 1.1) \text{GeV}^2, \quad (28)$$

which, albeit somewhat larger than the early value obtained by SVZ [1], appears to be more appropriate for future QCD sum rule analyses. It is unlikely that the gluon condensate suitable for light quark systems would be smaller than this value (as extracted primarily from charmonium systems). On the other hand, the quoted value on the quark condensate (as an order parameter of the phase transition), namely Eq. (13), may have an uncertainty as large as 20 %, should we use the instanton picture of the QCD vacuum [12] as a guide. Making use of the Ward-Takahashi identities associated with a specific nonlinear chiral transformation, Jacquot and Richert [13] obtain

$$m_{dyn}(\mu) \langle \bar{q}q \rangle = -\frac{1}{48\pi^2} \langle g^2(\mu) G^2 \rangle, \quad (29)$$

where  $m_{dyn}(\mu)$  is the dynamically generated quark mass at the scale  $\mu$ . This relation also follows from a dilute-gas approximation of the instanton QCD vacuum. Substituting the current values of the condensates into this relation, it is seen that the dynamically generated quark mass may be interpreted as the constituent quark mass. Since the relation (29) would fail badly with the current quark mass as the input, this result may be regarded as one of the reasons why the constituent or dynamically generated mass might be more appropriate for QCD sum rule studies.

It is possible to adopt Eq. (29) as a constraint among the quark mass, and the quark and gluon condensates. Such constraint leads to the gluon condensate which is a few times larger than the original SVZ value [1] but which is pretty much in line with the results from lattice simulations.

As indicated earlier, it is likely that the mixed quark-gluon condensate is smaller than the current value  $m_0^2 \sim 0.8 \text{ GeV}^2$  especially when the effective chiral quark theory is used. Lacking any further guideline on the matter, we recall the reduction factor in  $\alpha_s$  [6] and arbitrarily take  $m_0^2 \sim 0.4 \text{ GeV}^2$ , but caution that further justification is needed for coming up with a better value of  $m_0^2$ . The convergence property of the mass sum rule, Eq. (27), would be a suspect for  $m_0^2 \approx 0.8 \text{ GeV}^2$  but becomes very stable when a smaller value is assumed. In a sense we can make use of the nucleon mass sum rule to constrain the mixed quark-gluon condensate but the sum rule becomes so well behaved for sufficiently small  $m_0^2$  that the constraint loses its efficiency.

The logarithmic derivative of the LHS of Eq. (27) gives the prediction on the nucleon mass  $M$ , a prediction which turns out to be quite stable with respect to the Borel mass  $M_B$ . We obtain  $M = (0.94 \pm 0.02) \text{ GeV}$  for  $M_B = (1.10 \pm 0.05) \text{ GeV}$ . Substituting this value back into Eq. (27), we obtain  $\beta_N^2 = (0.177 \pm 0.005) \text{ GeV}^6$ . An additional error of at least  $\pm 0.4 \text{ GeV}$  for the predicted nucleon mass should be understood if we consider the uncertainty in the various condensates.

We turn our attention to the QCD sum rule for the strong  $\pi NN$  coupling [14, 9].

$$\begin{aligned}
& \frac{1}{4}M_B^6 L^{-4/9} E_2 - \frac{\chi_\pi a}{8} M_B^2 L^{2/9} E_1 - \frac{11}{96} \langle g_c^2 G^2 \rangle M_B^2 E_0 + \frac{1}{3} a^2 L^{4/9} \\
& + m a m_0^2 L^{-26/27} \left\{ \frac{3}{8} \ln \frac{M_B^2}{\mu^2} + \frac{49}{48} \right\} + \frac{1}{12} m_0^2 a^2 \frac{L^{-2/27}}{M_B^2} \\
& = g_{\pi q}^{-1} \{g_{\pi NN} + B M_B^2\} \beta_N^2 e^{-M^2/M_B^2}.
\end{aligned} \tag{30}$$

Note that the term  $B M_B^2$  is introduced to absorb some effect from excited states (with less singular pole behavior).

Numerically, we obtain  $g_{\pi NN} = -(14.8 \pm 0.7)$  for  $M_B = (1.10 \pm 0.05) \text{ GeV}$ , again a fairly stable result with respect to the Borel mass. An additional error of at least  $\pm 2.0$  should be understood in view of the uncertainties involved in the various condensates.

We wish to point out that about 50 % of the contribution in fact comes from the  $\chi_\pi a$  term, a nonperturbative effect. This result confirms the long-standing conviction that pion physics associated with nucleons is non-perturbative and can only be obtained by taking into account the effects related to the nontrivial vacuum. Experimentally, we have  $|g_{\pi NN}| \approx 13.5$ , which is in general agreement with the sum rule prediction.

It is indeed remarkable that the observed strong  $\pi NN$  coupling has been reproduced in QCD sum rules primarily because of the inclusion of the nonperturbative terms – the first term on the LHS is the leading perturbative contribution but it is not of much numerical importance. In other words, we have essentially shown that the large  $\pi NN$  coupling is nonperturbative in origin.

Finally, we may also turn our attention to the weak p.v. pion-nucleon coupling, which is defined as follows [15, 16, 17].

$$\mathcal{L}_{\pi NN}^{p.v.} = \frac{f_{\pi NN}}{\sqrt{2}} \bar{\psi} (\vec{\tau} \times \vec{\pi})_3 \psi. \tag{31}$$

Only charged pions can be emitted or absorbed. Here we modify the QCD sum rule which we obtained earlier [9] (and which has some pathological behavior) by multiplying both

sides (LHS and RHS) by the factor  $(p^2 - M^2)$  immediately before the Borel transform. This has helped to produce a QCD sum rule which is very well behaved.

$$\begin{aligned} & \frac{G_F \sin^2 \theta_W (\frac{17}{3} - \gamma)}{96\pi^2} \left[ (4M_B^{10} - M^2 M_B^8) L^{-4/9} E_3 \right. \\ & \quad \left. - (2M_B^8 - \frac{2}{3} M^2 M_B^6) \chi_\pi a L^{-4/9} E_2 - (M_B^6 - \frac{1}{2} M^2 M_B^4) m_0^\pi a E_1 L^{-4/9} \right] \\ & = g_{\pi q}^{-1} \{ f_{\pi NN} + B' M_B^2 \} \beta_N^2 2M^2 e^{-M^2/M_B^2}. \end{aligned} \quad (32)$$

This sum rule still suffers from the fact that the contribution from the gluon condensate diagrams is yet to be included; these diagrams are much more complicated to evaluate although they are of the same order or smaller than uncertainties of our calculation.

Numerically, we obtain  $f_{\pi NN} = (3.04 \pm 0.01) \times 10^{-7}$  for  $M_B = (1.10 \pm 0.05) \text{ GeV}$ , a prediction which is even more stable than the previous two sum rules (on the nucleon mass and the strong pion-nucleon coupling). The uncertainties in the condensates and in the terms which have been neglected amount to at least  $\pm 0.5 \times 10^{-7}$ . About 50 % of the contribution comes from the nonperturbative  $\chi_\pi a$  term.

Our prediction of  $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$  (at  $M_B \sim 1.0 \text{ GeV}$ ) is in good agreement with the value ( $\sim 2 \times 10^{-7}$ ) obtained by Adelberger and Haxton [15] from an overall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent experiment [18]. Once again, we see that the non-perturbative physics dictates in the prediction of the parity-violating weak  $\pi NN$  coupling, a typical case of nonleptonic weak interactions. Our results suggest the need to incorporate such non-perturbative effects into any systematic study of nonleptonic weak interactions. We suspect that many important issues, such as the  $\Delta I = \frac{1}{2}$  rule, in relation to nonleptonic weak interactions cannot be addressed in any meaningful manner without suitable incorporation of non-perturbative condensate effects.



## IV. Discussion and Summary

In light of the very encouraging results which we have obtained for the nucleon mass and the strong and weak  $\pi NN$  couplings, it is tempting to take seriously the proposal that the matching in QCD sum rules is done between the hadron world and the effective chiral quark theory [5,6]. In particular, non-perturbative QCD effects in a large number of physical processes involving Goldstone bosons may now be treated on the same footing.

The uncertainties involved in the various condensates pose some problem in the analysis of QCD sum rules. As a numerical indicator, we note that an increase of 10 % in the gluon condensate produces little change in the predicted nucleon mass  $M$  ( $-0.4$  %) and the p.v. pion-nucleon coupling  $f_{\pi NN}$  ( $-1.5$  %) but reduces the value of the strong pion-nucleon coupling  $|g_{\pi NN}|$  by 5.7 %. A reduction of  $m_0^2$  by 20 % (on the mixed quark-gluon condensate) produces the changes of  $-2.1$  %,  $-12.6$  %, and  $+15.1$  % respectively for  $M$ ,  $|g_{\pi NN}|$ , and  $f_{\pi NN}$ . On the other hand, a reduction of the quark condensate by 10 % increases  $M$  by 4 %, decreases  $|g_{\pi NN}|$  by 2.2 %, and reduces  $f_{\pi NN}$  by 12.4 %. These results enable us to assign some errors to our predictions cited in the previous section. Nevertheless, some room remains for an entirely different set of parameters. For instance, we may remark that a relativistic version of the effective chiral quark theory based on the Bethe-Salpeter equation (BSE) and Schwinger-Dyson equation (SDE), such as [19], could also lead to similar final numerical results. Therefore it is premature to take the present numerics as the credence towards only the nonrelativistic chiral quark model [6].

There are also many other important issues related to the proposal. For example, the adoption of the constituent quark mass could modify the existing QCD sum rule calculations, perhaps quite severely in some cases. Furthermore, pions may also be treated as “quantized fields”, leading to the concept of “pion condensation” in addition to the better known pion-loop corrections, should we take the effective chiral quark theory to the next level of sophistication. Finally, there are issues concerning the importance of the

instantons [12] when pions are involved.

First, we discuss briefly how Goldstone pions can be treated as “quantized fields”. This involves integration of the renormalization procedure as now widely adopted for chiral perturbation theory with the standard renormalization for quantized gauge field theories. This is not a trivial task which requires further studies. To a given order (in terms of, e.g., the number of pion loops), it might be possible to obtain the “renormalized” effective chiral quark theory which can then be used for mapping onto the world of hadrons. For the purpose of the present paper, however, we note that the problems which involve such renormalizations involve terms at least bi-linear in pion fields and thus are always of higher order in nature.

Next, we may discuss the changes when the constituent quark mass instead of the very small current quark mass is adopted. As a qualitative estimate, we approximate the first term of Eq. (24) by the first term of Eq. (16) – we neglect a term of order  $m^2/p^2$  (with  $p^2 \sim 1 \text{ GeV}^2$ ), a (10-20) % effect. It affects most of the existing QCD sum rule predictions only in quantitative detail, but not qualitatively. This point is also exemplified in detail in our analysis of the sum rules on the nucleon mass and the strong pion-nucleon coupling.

It is clear that the proposal of matching the world of hadrons to that of the effective chiral quark theory raises many interesting questions related to fundamentals in QCD sum rules. It is also clear that it would be well beyond the scope of the present paper to address many such questions in detail. It is hoped that the encouraging results which we have obtained in the present paper would help to motivate further investigations along this direction.

To sum up, we have investigated, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We have shown how the proposal yields predictions on induced condensates which reproduce the observed value of the strong  $\pi NN$  coupling constant. Our result indicate that the observed large  $\pi NN$  coupling comes

primarily from the nonperturbative induced-condensate effect. On the other hand, the prediction on the parity-violating weak  $\pi NN$  coupling,  $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$ , is in good agreement with the value ( $\sim 2 \times 10^{-7}$ ) obtained by Adelberger and Haxton [15] from an overall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent null experiment [18]. Altogether, we wish to argue that the proposal of matching onto a chirally broken effective theory should enable us to treat, in a quantitative and consistent manner, the various nonleptonic strong and weak processes involving Goldstone bosons.

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### References

1. M. A. Shifman, A.J. Vainshtein, and V.I. Zakharov, Nucl. Phys. **147**, 385, 448 (1979).
2. T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Clarendon Press, Oxford, 1984); T.-Y. Wu and W.-Y. P. Hwang, *Relativistic Quantum Mechanics and Quantum Fields* (World Scientific, Singapore, 1991).
3. B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); (E) **B191**, 591 (1981); V. M. Belyaev and B. L. Ioffe, Zh. Eksp. Teor. Fiz. **83**, 876 (1982) [Sov. Phys. JETP **56**, 493 (1982)].
4. V. M. Belyaev and Ya. I. Kogan, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 611 (1983) [JETP Lett. **37**, 730 (1983)]; C. B. Chiu, J. Pasupathy, and S.J. Wilson, Phys. Rev. **D32**, 1786 (1985); E. M. Henley, W.-Y. P. Hwang, and L.S. Kisslinger, Phys. Rev. **D46**, 431 (1992); Chinese J. Phys. (Taipei) **30**, 529 (1992).

5. S. Weinberg, Phys. Lett. **B251**, 288 (1990); Physica **96A**, 327 (1979).
6. H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings Publishing Co., Menlo Park, California, 1984); A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
7. J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982); Nucl. Phys. **B250**, 465 (1985).
8. B.L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591 (E) (1981); Z. Phys. **C18**, 67 (1983).
9. E. M. Henley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B 367**, 21 (1996).
10. K.-C. Yang, W-Y. P. Hwang, E.M. Henley, and L.S. Kisslinger, Phys. Rev. **D47**, 3001 (1993).
11. S. Narison, Phys. Lett. **B 387**, 162 (1996).
12. T. Schäfer and E.V. Shuryak, 1996 preprint hep-ph/9610451.
13. J.-L. Jacquot and J. Richert, Z. Phys. **C - Particles and Fields**, **56**, 201 (1992).
14. W-Y. P. Hwang, Ze-sen Yang, Y.S. Zhong, Z.N. Zhou, and Shi-lin Zhu, 1996 preprint hep-ph/9610412.
15. See, e.g., E.G. Adelberger and W.C. Haxton, Ann. Rev. Nucl. Part. Sci. **35**, 501 (1985); and J. Lang *et al.*, Phys. Rev. **34**, 1545 (1986).
16. B. Desplanques, J.F. Donoghue, and B.R. Holstein, Ann. Phys. (NY) **124**, 449 (1980).
17. E.M. Henley, Ann. Rev. Nucl. Sci. **19**, 367 (1969; Chinese J. Phys. **30**, 1 (1992).
18. C.A. Barnes *et al.* Phys. Rev. Lett. **40**, 840 (1978); H.C. Evans *et al.*, Phys. Rev. Lett. **55**, 791 (1985); Phys. Rev. **C35**, 1119 (1987); M. Bini *et al.*, Phys. Rev. Lett. **55**, 795 (1985); Phys. Rev. **C38**, 1195 (1988).
19. K. K. Gupta et al., Phys. Rev. **D42**, 1604 (1990).